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Three kinds of compact thin subwavelength cavity resonators containing left-handed media: rectangular, cylindrical, spherical

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In the present paper we investigate the restriction conditions for three kinds of cavity resonators (*i.e.*, the rectangular, cylindrical, spherical resonators). It is shown that the layer of materials with negative optical refractive indices can act as a phase compensator/conjugator, and thus by combining such a layer with another layer made of the regular medium one can obtain a so-called compact thin subwavelength cavity resonator.

Keywords: subwavelength cavity resonators, left-handed medium

I. INTRODUCTION

More recently, a kind of artificial composite metamaterials (the so-called *left-handed media*) having a frequency band where the effective permittivity and the effective permeability are simultaneously negative attracts considerable attention of many authors both experimentally and theoretically [1–5]. In 1967[1], Veselago first considered this peculiar medium and showed from Maxwellian equations that such media having negative simultaneously negative ϵ and μ exhibit a negative index of refraction, *i.e.*, $n = -\sqrt{\epsilon\mu}$ [6]. It follows from the Maxwell's curl equations that the phase velocity of light wave propagating inside this medium is pointed opposite to the direction of energy flow, that is, the Poynting vector and wave vector of electromagnetic wave would be antiparallel, *i.e.*, the vector $\mathbf{k}$, the electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$ form a left-handed system; thus Veselago referred to such materials as "left-handed" media, and correspondingly, the ordinary medium in which $\mathbf{k}$, $\mathbf{E}$ and $\mathbf{H}$ form a right-handed system may be termed the "right-handed" one. Other authors call this class of materials "negative-index media (NIM)" [8], "backward media (BWM)" [7], "double negative media (DNM)" and Veselago's media. There exist a number of peculiar electromagnetic and optical properties, for instance, many dramatically different propagation characteristics stem from the sign change of the optical refractive index and phase velocity, including reversal of both the Doppler shift and Cerenkov radiation, anomalous refraction, amplification of evanescent waves [9], unusual photon tunneling [10], modified spontaneous emission rates and even reversals of radiation pressure to radiation tension [1]. In experiments, this artificial negative electric permittivity media may be obtained by using the *array of long metallic wires* (ALMWs) [11], which simulates the plasma behavior at microwave frequencies, and the artificial negative magnetic permeability media may be built up by using small resonant metallic particles, *e.g.*, the *split ring resonators* (SRRs), with very high magnetic polarizability [12]. A combination of the two structures yields a left-handed medium. Recently, Shelby *et al.* reported their first experimental realization of this artificial composite medium, the permittivity and permeability of which have negative real parts [1]. One of the potential applications of negative refractive index materials is to fabricate the so-called "superlenses" (perfect lenses): specifically, a slab of such materials may has the power to focus all Fourier components of a 2D image, even those that do not propagate in a radiative manner [9,13].

Engheta suggested that a slab of metamaterial with negative electric permittivity and magnetic permeability (and hence negative optical refractive index) can act as a phase compensator/conjugator and, therefore, by combining such a slab with another slab fabricated from a conventional (ordinary) dielectric material one can, in principle, have a 1-D

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[1]Note that, in the literature, some authors mentioned the wrong year when Veselago suggested the *left-handed media*. They claimed that Veselago proposed or introduced the concept of *left-handed media* in 1968 or 1964. On the contrary, the true history is as follows: Veselago's excellent paper was first published in Russian in July, 1967 [Usp. Fiz. Nauk **92**, 517-526 (1967)]. This original paper was translated into English by W.H. Furry and published again in 1968 in the journal of Sov. Phys. Usp. [6]. Unfortunately, Furry stated erroneously in his English translation that the original version of Veselago' work was first published in 1964.

cavity resonator whose dispersion relation may not depend on the sum of thicknesses of the interior materials filling this cavity, but instead it depends on the ratio of these thicknesses. Namely, one can, in principle, conceptualize a 1-D compact, subwavelength, thin cavity resonator with the total thickness far less than the conventional $\frac{\lambda}{2}$ [14].

Engheta's idea for the 1-D compact, subwavelength, thin cavity resonator is the two-layer rectangular structure (the left layer of which is assumed to be a conventional lossless dielectric material with permittivity and permeability being positive numbers, and the right layer is taken to be a lossless metamaterial with negative permittivity and permeability) sandwiched between the two reflectors (*e.g.*, two perfectly conducting plates) [14]. For the pattern of the 1-D subwavelength cavity resonator readers may be referred to the figures of reference [14]. Engheta showed that with the appropriate choice of the ratio of the thicknesses d_1 to d_2, the phase acquired by the incident wave at the left (entrance) interface to be the same as the phase at the right (exit) interface, essentially with no constraint on the total thickness of the structure. The mechanism of this effect may be understood as follows: as the planar electromagnetic wave exits the first slab, it enters the rectangular slab of metamaterial and finally it leaves this second slab. In the first slab, the direction of the Poynting vector is parallel to that of phase velocity, and in the second slab, however, these two vectors are antiparallel with each other. Thus the wave vector k_2 is therefore in the opposite direction of the wave vector k_1. So the total phase difference between the front and back faces of this two-layer rectangular structure is $k_1d_1 - |k_2|d_2$ [14]. Therefore, whatever phase difference is developed by traversing the first rectangular slab, it can be decreased and even cancelled by traversing the second slab. If the ratio of d_1 and d_2 is chosen to be $\frac{d_1}{d_2} = \frac{|k_2|}{k_1}$, then the total phase difference between the front and back faces of this two-layer structure becomes zero (*i.e.*, the total phase difference is not $2n\pi$, but instead of zero) [14]. As far as the properties and phenomena in the subwavelength cavity resonators is concerned, Tretyakov *et al.* investigated the evanescent modes stored in cavity resonators with backward-wave slabs [15].

II. A RECTANGULAR SLAB 1-D THIN SUBWAVELENGTH CAVITY RESONATOR

To consider the 1-D wave propagation in a compact, subwavelength, thin cavity resonator, we first take into account a slab cavity of three-layer structure, where the regions 1 and 2 are located on the left- and right- handed sides, and the plasmon-type medium (or a superconductor material) is between the regions 1 and 2. The above three-layer structure is assumed to be sandwiched between the two reflectors (or two perfectly conducting plates) [14]. Assume that the wave vector of the electromagnetic wave is parallel to the third component of Cartesian coordinate. The electric and magnetic fields in the region 1 (with the permittivity being ϵ_1 and the permeability being μ_1) are written in the form

$$E_{x1} = E_{01}\sin\left(n_1k_0z\right), \quad H_{y1} = \frac{n_1k_0}{i\omega\mu_1}E_{01}\cos\left(n_1k_0z\right) \quad (0 \le z \le d_1), \tag{2.1}$$

where k_0 stands for the wave vector of the electromagnetic wave under consideration in the free space, *i.e.*, $k_0 = \frac{\omega}{c}$, and in the region 2, where $d_1 + a \le z \le d_1 + d_2 + a$, the electric and magnetic fields are of the form

$$E_{x2} = E_{02}\sin\left[n_2k_0\left(z - d_1 - d_2 - a\right)\right], \quad H_{y2} = \frac{n_2k_0}{i\omega\mu_2}E_{02}\cos\left[n_2k_0\left(z - d_1 - d_2 - a\right)\right], \tag{2.2}$$

where d_1, d_2 and a denote the thicknesses of the regions 1, 2 and the plasmon (or superconducting) region, respectively. The subscripts 1 and 2 in the present paper denote the physical quantities in the regions 1 and 2. Note that here the optical refractive indices n_1 and n_2 are defined to be $n_1 = \sqrt{\epsilon_1\mu_1}$ and $n_2 = \sqrt{\epsilon_2\mu_2}$. Although in the present paper we will consider the wave propagation in the negative refractive index media, the choice of the signs for n_1 and n_2 will be irrelevant in the final results. So, we choose the plus signs for n_1 and n_2 no matter whether the materials 1 and 2 are of left-handedness or not. The choice of the solutions presented in (2.1) and (2.2) guarantees the satisfaction of the boundary conditions at the perfectly conducting plates at $z = 0$ and $z = d_1 + d_2 + a$.

The electric and magnetic fields in the plasmon (or superconducting) region (with the resonant frequency being $\omega_{\rm p}$) take the form

$$E_{xs} = A\exp(\beta z) + B\exp(-\beta z), \quad H_{ys} = \frac{\beta}{i\omega}\left[A\exp(\beta z) - B\exp(-\beta z)\right], \tag{2.3}$$

where the subscript s represents the quantities in the plasmon (or superconducting) region, and $\beta = \frac{\sqrt{\omega_{\rm p}^2 - \omega^2}}{c}$.

To satisfy the boundary conditions

$$E_{x1}|_{z=d_1} = E_{xs}|_{z=d_1}, \quad H_{y1}|_{z=d_1} = H_{ys}|_{z=d_1} \tag{2.4}$$

at the interface ($z = d_1$) between the region 1 and the plasmon region, we should have

$$E_{01}\sin(n_1k_0d_1) = A\exp(\beta d_1) + B\exp(-\beta d_1), \quad \frac{n_1k_0}{\beta\mu_1}E_{01}\cos(n_1k_0d_1) = A\exp(\beta d_1) - B\exp(-\beta d_1). \tag{2.5}$$

It follows that the parameters A and B in Eq.(2.3) are given as follows

$$A = \frac{1}{2}\exp(-\beta d_1)E_{01}\left[\sin(n_1k_0d_1) + \frac{n_1k_0}{\beta\mu_1}\cos(n_1k_0d_1)\right],$$
$$B = -\frac{1}{2}\exp(\beta d_1)E_{01}\left[\frac{n_1k_0}{\beta\mu_1}\cos(n_1k_0d_1) - \sin(n_1k_0d_1)\right]. \tag{2.6}$$

In the similar fashion, to satisfy the boundary conditions

$$E_{x2}|_{z=d_1+a} = E_{xs}|_{z=d_1+a}, \quad H_{y2}|_{z=d_1+a} = H_{ys}|_{z=d_1+a} \tag{2.7}$$

at the interface ($z = d_1 + a$) between the region 2 and the plasmon region, one should arrive at

$$E_{02}\sin(-n_2k_0d_2) = A\exp[\beta(d_1+a)] + B\exp[-\beta(d_1+a)],$$
$$\frac{n_2k_0}{\beta\mu_2}E_{02}\cos(-n_2k_0d_2) = A\exp[\beta(d_1+a)] - B\exp[-\beta(d_1+a)]. \tag{2.8}$$

It follows that the parameters A and B in Eq.(2.3) are given as follows

$$A = -\frac{1}{2}E_{02}\exp[-\beta(d_1+a)]\left[\sin(n_2k_0d_2) - \frac{n_2k_0}{\beta\mu_2}\cos(n_2k_0d_2)\right],$$
$$B = -\frac{1}{2}\exp[-\beta(d_1+a)]E_{02}\left[\frac{n_2k_0}{\beta\mu_2}\cos(n_2k_0d_2) + \sin(n_2k_0d_2)\right]. \tag{2.9}$$

Thus, according to Eq.(2.6) and (2.9), we can obtain the following conditions

$$E_{01}\left[\sin(n_1k_0d_1) + \frac{n_1k_0}{\beta\mu_1}\cos(n_1k_0d_1)\right] + E_{02}\exp(-\beta a)\left[\sin(n_2k_0d_2) - \frac{n_2k_0}{\beta\mu_2}\cos(n_2k_0d_2)\right] = 0,$$
$$E_{01}\left[\frac{n_1k_0}{\beta\mu_1}\cos(n_1k_0d_1) - \sin(n_1k_0d_1)\right] - E_{02}\exp(\beta a)\left[\frac{n_2k_0}{\beta\mu_2}\cos(n_2k_0d_2) + \sin(n_2k_0d_2)\right] = 0. \tag{2.10}$$

In order to have a nontrivial solution, *i.e.*, to have $E_{01} \neq 0$ and $E_{02} \neq 0$, the determinant in Eq.(2.10) must vanish. Thus we obtain the following restriction condition

$$[\exp(\beta a) + \exp(-\beta a)]\left[\frac{n_1}{\mu_1}\tan(n_2k_0d_2) + \frac{n_2}{\mu_2}\tan(n_1k_0d_1)\right]$$
$$+\frac{\beta}{k_0}[\exp(\beta a) - \exp(-\beta a)]\left[\tan(n_1k_0d_1)\tan(n_2k_0d_2) + \frac{n_1n_2k_0^2}{\beta^2\mu_1\mu_2}\right] = 0 \tag{2.11}$$

for the electromagnetic wave in the three-layer-structure rectangular cavity.

If the thickness, a, of the plasmon region is vanishing (*i.e.*, there exists no plasmon region), then the restriction equation (2.11) is simplified to

$$\frac{n_1}{\mu_1}\tan(n_2k_0d_2) + \frac{n_2}{\mu_2}\tan(n_1k_0d_1) = 0. \tag{2.12}$$

In what follows we will demonstrate why the introduction of left-handed media will give rise to the novel design of the compact thin cavity resonator. If the material in region 1 is a regular medium while the one in region 2 is the left-handed medium, it follows that

$$\frac{\tan(n_1k_0d_1)}{\tan(n_2k_0d_2)} = \frac{-n_1\mu_2}{n_2\mu_1}. \tag{2.13}$$

According to Engheta [14], this relation does not show any constraint on the sum of thicknesses of d_1 and d_2. It rather deals with the ratio of tangent of these thicknesses (with multiplicative constants). If we assume that ω, d_1 and d_2 are chosen such that the small-argument approximation can be used for the tangent function, the above relation can be simplified as

$$\frac{d_1}{d_2} \simeq -\frac{\mu_2}{\mu_1}. \tag{2.14}$$

This relation shows even more clearly how d_1 and d_2 should be related in order to have a nontrivial 1-D solution with frequency ω for this cavity. So conceptually, what is constrained here is $\frac{d_1}{d_2}$, not $d_1 + d_2$. Therefore, in principle, one can have a thin subwavelength cavity resonator for a given frequency [14].

So, one of the most exciting ideas is the possibility to design the so-called compact thin subwavelength cavity resonators. It was shown that a pair of plane waves travelling in the system of two planar slabs positioned between two metal planes can satisfy the boundary conditions on the walls and on the interface between two slabs even for arbitrarily thin layers, provided that one of the slabs has negative material parameters [15].

In the following let us take account of two interesting cases:

(i) If $\beta a \to 0$, then the restriction equation (2.11) is simplified to

$$\frac{n_1}{\mu_1}\tan(n_2k_0d_2) + \frac{n_2}{\mu_2}\tan(n_1k_0d_1) + \frac{2\beta^2 a}{k_0}\left[\tan(n_1k_0d_1)\tan(n_2k_0d_2) + \frac{n_1n_2k_0^2}{\beta^2\mu_1\mu_2}\right] = 0, \tag{2.15}$$

which yields

$$\tan(n_1k_0d_1) = \frac{k_0}{\beta}\frac{n_1}{\mu_1}, \quad \tan(n_2k_0d_2) = -\frac{k_0}{\beta}\frac{n_2}{\mu_2}. \tag{2.16}$$

If both $n_1k_0d_1$ and $n_2k_0d_2$ are very small, then one can arrive at $d_1 \doteq \frac{1}{\beta\mu_1}$, $d_2 \doteq -\frac{1}{\beta\mu_2}$ from Eq.(2.16), which means that the thicknesses d_1 and d_2 depend upon the plasmon parameter β.

(ii) If the resonant frequency ω_{p} is very large (and hence β), then it follows from Eq.(2.10) and (2.11) that

$$\frac{n_1}{\mu_1}\tan(n_2k_0d_2) = 0, \quad \frac{n_2}{\mu_2}\tan(n_1k_0d_1) = 0, \tag{2.17}$$

namely, regions 1 and 2 are isolated from each other., which is a result familiar to us.

In conclusion, as was shown by Engheta, it is possible that when one of the slab has a negative permeability, electromagnetic wave in two adjacent slabs bounded by two metal walls can satisfy the boundary conditions even if the distance between the two walls is much smaller than the wavelength [14].

III. A CYLINDRICAL THIN SUBWAVELENGTH CAVITY RESONATOR

Here we will consider the restriction equation for a cylindrical cavity to be a thin subwavelength cavity resonator containing left-handed media. It is well known that the Helmholtz equation $\nabla^2\mathbf{E} + k^2\mathbf{E} = 0$ in an axially symmetric cylindrical cavity (with the 2-D polar coordinates ρ and φ) can be rewritten as

$$\begin{aligned}
&\nabla^2 E_\rho - \frac{1}{\rho^2}E_\rho - \frac{2}{\rho^2}\frac{\partial E_\varphi}{\partial\varphi} + k^2E_\rho = 0,\\
&\nabla^2 E_\varphi - \frac{1}{\rho^2}E_\varphi + \frac{2}{\rho^2}\frac{\partial E_\rho}{\partial\varphi} + k^2E_\varphi = 0,\\
&\nabla^2 E_z + k^2E_z = 0.
\end{aligned} \tag{3.1}$$

One can obtain the electromagnetic field distribution, E_ρ, E_φ and H_ρ, H_φ, in the above axially symmetric cylindrical cavity via Eq.(3.1). But here we will adopt another alternative way to get the solutions of electromagnetic fields in the cylindrical cavity. If the electromagnetic fields are time-harmonic, *i.e.*, $\mathbf{E}(\rho,\varphi,z,t) = \vec{\mathcal{E}}(\rho,\varphi)\exp[i(hz-kct)]$ and $\mathbf{H}(\rho,\varphi,z,t) = \vec{\mathcal{H}}(\rho,\varphi)\exp[i(hz-kct)]$, then it follows from Maxwell equations that

$$
\begin{aligned}
-ikc\mathcal{E}_\rho &= \frac{1}{\epsilon}\left(\frac{1}{\rho}\frac{\partial\mathcal{H}_z}{\partial\varphi} - ih\mathcal{H}_\varphi\right), \\
-ikc\mathcal{E}_\varphi &= \frac{1}{\epsilon}\left(ih\mathcal{H}_\rho - \frac{\partial\mathcal{H}_z}{\partial\rho}\right), \\
-ikc\mathcal{E}_z &= \frac{1}{\epsilon}\left(\frac{\partial\mathcal{H}_\varphi}{\partial\rho} + \frac{1}{\rho}\mathcal{H}_\varphi - \frac{1}{\rho}\frac{\partial\mathcal{H}_\rho}{\partial\varphi}\right),
\end{aligned}
\tag{3.2}
$$

and

$$
\begin{aligned}
ikc\mathcal{H}_\rho &= \frac{1}{\mu}\left(\frac{1}{\rho}\frac{\partial\mathcal{E}_z}{\partial\varphi} - ih\mathcal{E}_\varphi\right), \\
ikc\mathcal{H}_\varphi &= \frac{1}{\mu}\left(ih\mathcal{E}_\rho - \frac{\partial\mathcal{E}_z}{\partial\rho}\right), \\
ikc\mathcal{H}_z &= \frac{1}{\mu}\left(\frac{\partial\mathcal{E}_\varphi}{\partial\rho} + \frac{1}{\rho}\mathcal{E}_\varphi - \frac{1}{\rho}\frac{\partial\mathcal{E}_\rho}{\partial\varphi}\right).
\end{aligned}
\tag{3.3}
$$

Thus it is demonstrated that the electromagnetic fields $\mathcal{E}_\rho$, $\mathcal{E}_\varphi$ and $\mathcal{H}_\rho$, $\mathcal{H}_\varphi$ can be expressed in terms of $\mathcal{E}_z$ and $\mathcal{H}_z$, *i.e.*,

$$
\mathcal{E}_\rho = \frac{i}{k^2-h^2}\left(h\frac{\partial\mathcal{E}_z}{\partial\rho} + \frac{k^2}{\epsilon\rho}\frac{\partial\mathcal{H}_z}{\partial\varphi}\right), \quad \mathcal{E}_\varphi = \frac{i}{k^2-h^2}\left(h\frac{1}{\rho}\frac{\partial\mathcal{E}_z}{\partial\varphi} - \frac{k^2}{\epsilon}\frac{\partial\mathcal{H}_z}{\partial\rho}\right),
\tag{3.4}
$$

and

$$
\mathcal{H}_\rho = \frac{i}{k^2-h^2}\left(h\frac{\partial\mathcal{H}_z}{\partial\rho} - \frac{k^2}{\mu\rho}\frac{\partial\mathcal{E}_z}{\partial\varphi}\right), \quad \mathcal{H}_\varphi = \frac{i}{k^2-h^2}\left(h\frac{1}{\rho}\frac{\partial\mathcal{H}_z}{\partial\varphi} + \frac{k^2}{\mu}\frac{\partial\mathcal{E}_z}{\partial\rho}\right).
\tag{3.5}
$$

As an illustrative example, in what follows, we will consider only the TM wave (*i.e.*, $\mathcal{H}_z = 0$) in the axially symmetric double-layer cylindrical thin subwavelength cavity resonator. Assume that the permittivity, permeability and radius of media in the interior and exterior layers of this double-layer cavity resonator are ϵ_1, μ_1, R_1 and ϵ_2, μ_2, R_2, respectively. According to the Helmholtz equation (with the boundary material being the perfectly conducting medium, $\mathcal{E}_{2z}|_{R_2} = 0$), we can obtain $E_{1z} = J_m\left(\sqrt{k_1^2-h_1^2}\rho\right)\left\{\begin{array}{c}\cos m\varphi \\ \sin m\varphi\end{array}\right\}$ and $E_{2z} = \left[AJ_m\left(\sqrt{k_2^2-h_2^2}\rho\right) + BN_m\left(\sqrt{k_2^2-h_2^2}\rho\right)\right]\left\{\begin{array}{c}\cos m\varphi \\ \sin m\varphi\end{array}\right\}$. Thus it follows from Eq.(3.4) and (3.5) that the electromagnetic fields in both interior and exterior layers are of the form

$$
\begin{aligned}
E_{1\rho} &= \frac{ih_1}{\sqrt{k_1^2-h_1^2}}J'_m\left(\sqrt{k_1^2-h_1^2}\rho\right)\left\{\begin{array}{c}\cos m\varphi \\ \sin m\varphi\end{array}\right\}, \\
E_{1\varphi} &= \frac{imh_1}{\left(k_1^2-h_1^2\right)\rho}J_m\left(\sqrt{k_1^2-h_1^2}\rho\right)\left\{\begin{array}{c}\sin m\varphi \\ -\cos m\varphi\end{array}\right\}, \\
E_{1z} &= J_m\left(\sqrt{k_1^2-h_1^2}\rho\right)\left\{\begin{array}{c}\cos m\varphi \\ \sin m\varphi\end{array}\right\},
\end{aligned}
\tag{3.6}
$$

$$
\begin{aligned}
\mathcal{H}_{1\rho} &= \frac{imk_1^2}{\left(k_1^2-h_1^2\right)\rho}\frac{1}{\mu_1}J_m\left(\sqrt{k_1^2-h_1^2}\rho\right)\left\{\begin{array}{c}-\sin m\varphi \\ \cos m\varphi\end{array}\right\}, \\
\mathcal{H}_{1\varphi} &= \frac{ik_1^2}{\sqrt{k_1^2-h_1^2}}\frac{1}{\mu_1}J'_m\left(\sqrt{k_1^2-h_1^2}\rho\right)\left\{\begin{array}{c}\cos m\varphi \\ \sin m\varphi\end{array}\right\}, \\
\mathcal{H}_{1z} &= 0,
\end{aligned}
\tag{3.7}
$$

and

$$
\begin{aligned}
E_{2\rho} &= \frac{ih_2}{\sqrt{k_2^2-h_2^2}}\left[AJ'_m\left(\sqrt{k_2^2-h_2^2}\rho\right) + BN'_m\left(\sqrt{k_2^2-h_2^2}\rho\right)\right]\left\{\begin{array}{c}\cos m\varphi \\ \sin m\varphi\end{array}\right\}, \\
E_{2\varphi} &= \frac{imh_2}{\left(k_2^2-h_2^2\right)\rho}\left[AJ_m\left(\sqrt{k_2^2-h_2^2}\rho\right) + BN_m\left(\sqrt{k_2^2-h_2^2}\rho\right)\right]\left\{\begin{array}{c}\sin m\varphi \\ -\cos m\varphi\end{array}\right\}, \\
E_{2z} &= \left[AJ_m\left(\sqrt{k_2^2-h_2^2}\rho\right) + BN_m\left(\sqrt{k_2^2-h_2^2}\rho\right)\right]\left\{\begin{array}{c}\cos m\varphi \\ \sin m\varphi\end{array}\right\},
\end{aligned}
\tag{3.8}
$$

$$\mathcal{H}_{2\rho}=\frac{imk_2^2}{\left(k_2^2-h_2^2\right)\rho}\frac{1}{\mu_2}\left[AJ_m\left(\sqrt{k_2^2-h_2^2}\rho\right)+BN_m\left(\sqrt{k_2^2-h_2^2}\rho\right)\right]\left\{\begin{array}{c}-\sin m\varphi\\ \cos m\varphi\end{array}\right\},$$
$$\mathcal{H}_{2\varphi}=\frac{ik_2^2}{\sqrt{k_2^2-h_2^2}}\frac{1}{\mu_2}\left[AJ'_m\left(\sqrt{k_2^2-h_2^2}\rho\right)+BN'_m\left(\sqrt{k_2^2-h_2^2}\rho\right)\right]\left\{\begin{array}{c}\cos m\varphi\\ \sin m\varphi\end{array}\right\},$$
$$\mathcal{H}_{2z}=0, \tag{3.9}$$

where J_m and N_m denote Bessel functions and Norman functions, respectively, and $k_1=\sqrt{\epsilon_1\mu_1}\frac{\omega}{c}$, $k_2=\sqrt{\epsilon_2\mu_2}\frac{\omega}{c}$.

By using the boundary conditions $\mathcal{E}_{2z}|_{\rho=R_2}=0$, $\mathcal{E}_{2\varphi}|_{\rho=R_2}=0$ (due to the perfectly conducting medium at the boundary $\rho=R_2$), one can determine the relationship between A and B, *i.e.*,

$$B=-A\frac{J_m\left(\sqrt{k_2^2-h_2^2}R_2\right)}{N_m\left(\sqrt{k_2^2-h_2^2}R_2\right)}. \tag{3.10}$$

By using the boundary conditions $\mathcal{E}_{1z}|_{R1}=\mathcal{E}_{2z}|_{R1}$, $\mathcal{E}_{1\varphi}|_{R1}=\mathcal{E}_{2\varphi}|_{R1}$, one can obtain

$$J_m\left(\sqrt{k_1^2-h_1^2}R_1\right)=AJ_m\left(\sqrt{k_2^2-h_2^2}R_1\right)+BN_m\left(\sqrt{k_2^2-h_2^2}R_1\right),$$
$$\frac{h_1}{k_1^2-h_1^2}J_m\left(\sqrt{k_1^2-h_1^2}R_1\right)=\frac{h_2}{k_2^2-h_2^2}\left[AJ_m\left(\sqrt{k_2^2-h_2^2}R_1\right)+BN_m\left(\sqrt{k_2^2-h_2^2}R_1\right)\right]. \tag{3.11}$$

The second equation in Eq.(3.11) is employed to determine the relation between h_1 and h_2. With the help of the boundary conditions $\mathcal{H}_{1z}|_{R1}=\mathcal{H}_{2z}|_{R1}$, $\mathcal{H}_{1\varphi}|_{R1}=\mathcal{H}_{2\varphi}|_{R1}$, one can arrive at

$$\frac{n_1}{\sqrt{k_1^2-h_1^2}}\frac{1}{\mu_1}J'_m\left(\sqrt{k_1^2-h_1^2}R_1\right)=\frac{n_2}{\sqrt{k_2^2-h_2^2}}\frac{1}{\mu_2}\left[AJ'_m\left(\sqrt{k_2^2-h_2^2}R_1\right)+BN'_m\left(\sqrt{k_2^2-h_2^2}R_1\right)\right]. \tag{3.12}$$

Combination of the first equation in Eq.(3.11) and (3.12), we have

$$\frac{n_1}{\mu_1\sqrt{k_1^2-h_1^2}}\frac{J'_m\left(\sqrt{k_1^2-h_1^2}R_1\right)}{J_m\left(\sqrt{k_1^2-h_1^2}R_1\right)}=\frac{n_2}{\mu_2\sqrt{k_2^2-h_2^2}}\frac{AJ'_m\left(\sqrt{k_2^2-h_2^2}R_1\right)+BN'_m\left(\sqrt{k_2^2-h_2^2}R_1\right)}{AJ_m\left(\sqrt{k_2^2-h_2^2}R_1\right)+BN_m\left(\sqrt{k_2^2-h_2^2}R_1\right)}, \tag{3.13}$$

which may be viewed as the restriction condition for the cylindrical cavity resonator. If we choose a typical case with $h_1=h_2=0$, then the obtained restriction condition is simplified to be

$$\frac{1}{\mu_1}\frac{J'_m\left(k_1R_1\right)}{J_m\left(k_1R_1\right)}=\frac{1}{\mu_2}\frac{AJ'_m\left(k_2R_1\right)+BN'_m\left(k_2R_1\right)}{AJ_m\left(k_2R_1\right)+BN_m\left(k_2R_1\right)}. \tag{3.14}$$

Substitution of the relation (3.10) into (3.14) yields

$$\frac{1}{\mu_1}\frac{J'_m\left(k_1R_1\right)}{J_m\left(k_1R_1\right)}=\frac{1}{\mu_2}\frac{J'_m\left(k_2R_1\right)N_m\left(k_2R_2\right)-J_m\left(k_2R_2\right)N'_m\left(k_2R_1\right)}{J_m\left(k_2R_1\right)N_m\left(k_2R_2\right)-J_m\left(k_2R_2\right)N_m\left(k_2R_1\right)}. \tag{3.15}$$

Eq.(3.15) is just the simplified restriction condition for the cylindrical cavity resonator.

Similar to the analysis presented in Sec. I, it is readily shown that by introducing the left-handed media such cylindrical cavity will also act as a compact thin subwavelength resonator. The discussion on this subject will not be performed further in the present paper.

IV. A SPHERICAL THIN SUBWAVELENGTH CAVITY RESONATOR

Here we will consider briefly the restriction equation for a spherical cavity to be a thin subwavelength cavity resonator containing left-handed media. Assume that the permittivity, permeability and radius of media in the interior and exterior layers of this double-layer cavity resonator are ϵ_1, μ_1, ρ_1 and ϵ_2, μ_2, ρ_2, respectively, and that the boundary medium at $\rho=\rho_2$ is the perfectly conducting material. Note that here the functions, symbols and

quantities are adopted in the paper [5], *e.g.*, N_1, N_2 denote the refractive indices of interio and exterior layers, and j_n and $h_n^{(1)}$ are the spherical Bessel and Hankel functions.

According to the paper [5], it follows from the boundary condition at $\rho=\rho_2$ that

$$a_n^m j_n(N_2\rho_2)+a_n^{\bar{m}}h_n^{(1)}(N_2\rho_2)=0,$$
$$b_n^m\left[N_2\rho_2 j_n(N_2\rho_2)\right]'+b_n^{\bar{m}}\left[N_2\rho_2 h_n^{(1)}(N_2\rho_2)\right]'=0. \tag{4.1}$$

The roles of Eqs.(4.1) is to determine the Mie coefficients $a_n^{\bar{m}}$ and $b_n^{\bar{m}}$ in terms of a_n^m and b_n^m.

At the boundary $\rho=\rho_1$, it follows from the boundary condition $\mathbf{i}_1\times\mathbf{E}_{\rm m}=\mathbf{i}_1\times\mathbf{E}_{\rm t}$ that one can obtain

$$a_n^m j_n(N_2\rho_1)+a_n^{\bar{m}}h_n^{(1)}(N_2\rho_1)=a_n^t j_n(N_1\rho_1),$$
$$N_1 b_n^m\left[N_2\rho_1 j_n(N_2\rho_1)\right]'+N_1 b_n^{\bar{m}}\left[N_2\rho_1 h_n^{(1)}(N_2\rho_1)\right]'=N_2 b_n^t\left[N_1\rho_1 j_n(N_1\rho_1)\right]'. \tag{4.2}$$

The role of the first and second equations in Eq.(4.2) is to obtain the expressions for the Mie coefficients a_n^t and b_n^t in terms of a_n^m and b_n^m, respectively.

In the same manner, at the boundary $\rho=\rho_1$, it follows from the boundary condition $\mathbf{i}_1\times\mathbf{H}_{\rm m}=\mathbf{i}_1\times\mathbf{H}_{\rm t}$ that one can obtain

$$\mu_1\left\{a_n^m\left[N_2\rho_1 j_n(N_2\rho_1)\right]'+a_n^{\bar{m}}\left[N_2\rho_1 h_n^{(1)}(N_2\rho_1)\right]'\right\}=\mu_2 a_n^t\left[N_1\rho_1 j_n(N_1\rho_1)\right]',$$
$$N_2\mu_1\left[b_n^m j_n(N_2\rho_1)+b_n^{\bar{m}}h_n^{(1)}(N_2\rho_1)\right]=N_1\mu_2 b_n^t j_n(N_1\rho_1). \tag{4.3}$$

Insertion of the expressions for the Mie coefficients $a_n^{\bar{m}}$ and $b_n^{\bar{m}}$ and a_n^t and b_n^t in terms of a_n^m and b_n^m obtained by Eqs.(4.1) and (4.2) into Eqs.(4.3) will lead to a set of equations of the Mie coefficients a_n^m and b_n^m. In order to have a nontrivial solutions of a_n^m and b_n^m, the determinant in Eqs.(4.3) must vanish. Thus we will obtain a restriction condition for the spherical cavity resonator.

By analogy with the analysis presented in Sec. I, it is easily verified that by involving the left-handed media such spherical cavity will also serve as a compact thin subwavelength resonator. In a word, the possibility to satisfy the boundary conditions for small distances between metal plates is based on the fact that plane waves in Veselago media are backward waves, meaning that the phase shift due to propagation in a usual slab can be compensated by a negative phase shift inside a backward-wave slab [14,15]. We will not discuss further this topic in this paper.

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